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ABSTRACT

A nonequivalent control group design study, with a math unit on ratio and proportion as the treatment, was conducted to examine the validity of some written tasks as a measure of formal reasoning. Subjects in the study were 68 fifth grade students who enrolled in two classes in a small urban school. The experimental group learned a 12-lesson math unit, which dealt with techniques to solve written problems of ratio and proportion. During this period the control group continued to study the regular mathematics curriculum, which at that time was decimal fractions. Pre- and posttests included ratio and proportion tasks from different group tests of formal reasoning. Findings showed that school learning did not improve students' ability to solve x/y=c type tasks, but did not improve their ability to solve tasks of multiplicative compensation. Since no improvement was observed in the ability of students in the control group to solve any of the above tasks, it was concluded that performance of some of the tasks that are used in group tests of formal reasoning are highly dependent on specific previous learning, and as such they should not be used as a sole indicator of formal reasoning. (Author/TW)

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Proportional Reasoning Tasks As a Measure of Formal Reasoning Ability

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Proportional Reasoning Tasks as a Measure

of Formal Reasoning Ability

Abstract

Nonequivalent control group design study, with a math unit of ratio and proportion as the treatment, served to examine the validity of some written tasks as a measure of formal reasoning. Subjects in the study were 68 fifth grade students who enrolled in two classes in a small urban school. The experimental group learned a 12-lesson math unit, which dealt with techniques to solve written problems of ratio and proportion. During this period the control group continued to study the regular math curriculum, which at that time was decimal fractions. Pre- and posttests included ratio and proportion tasks from different group tests of formal reasoning. Findings showed that school learning has improved students' ability to solve x/y=c type tasks, but did not improve their ability to solve tasks of multiplicative compensation. Since no improvement was observed in the ability of students in the control group to solve any of the above tasks, it can be concluded that performance of some of the tasks that are used in group tests of formal reasoning is highly dependent on specific previous learning, and as such they should not be used as a sole indicator of formal reasoning.

Proportional Reasoning Tasks as a Measure of Formal Reasoning Ability

Introduction

Studies in Science eduction have found that a great part of adolescent students are not at a formal operational stage (For a comprehensive review see Lawson, 1985). Many of these studies based their conclusions on findings which were obtained by testing students with paper and pencil tests only (e.g. Karplus at al, 1975; Onslow, 1976; Renner & Paske, 1977; Thornton & Fuller, 1980)

Karplus et al (1975), in their comparative study of proportional reasoning and control of variables in seven countries, have shown the significant effect of school learning on performance of formal reasoning tasks. In their research it was difficult to separate the effect of school type from the effect of the socioeconomic status of the students.

Study by Shemesh (1986) found that participation in high school science courses is highly correlated with successful performance of fomal reasoning tasks. In this study also, it was difficult to separate the effect of school learning from the effect of students' fluid intelligence. Nevertheless, from both studies and others, it can be concluded that performance of written tasks of formal reasoning is influenced by previous learing. If this situation is limited only to assessment of adolescent reasoning, it does not invalidate the written tests,

but if the same situation is found with young students, ones that are not assumed to have formal reasoning ability, it will question the validity of these tasks as measure of formatl reasoning.

Tasks in the written tests of formal reasoning are varied and different, but alsmost all of these tests contain at least one task of proportional reasoning, usually more than one.
Are all these tasks measuring the same congnitive structure? Are all of them a valid measure of formal reasoning?

The objective of this study was to anser the above questions, through an intervention study. For this purpose a pre-posttest nonequivalent control group design study was conducted, with three weeks of teaching ratio and proportion as the treatment.

In order to restrict achievement to learning, the study should have been done with young students, who were still holding concrete patterns of reasoning. The assumption behind this design is that three weeks is too short a time for a congnitive developmental change to occur, although it is long enough for 5th grade students to learn how to solve ratio and proportion problems. So, tasks that fifth grade students fail to solve before the treatment and solve successfully after it, can hardly be considered as a valid measure of formal reasoning.

Method

Subjects in this study were 68 fifth grade students (36 boys and 32 girls, average age 10 years and 9 months), who enrolled in two

classes in a small urban elementary school in Israel. Classes were randomly assigned to be experimental or control group.

Procedure

Pre-posttest nonequivalent control group design (Campbell and Stanley, 1963), with performance of proportional reasoning tasks as the dependent variable and three weeks of learning ratio and proportions as the treatment, was the basic design of this study.

Treatment

The experimental group learned how to solve different kinds of ratio and proportion problems during three weeks (12 lessons). At the same time the control group learned how to add and substract decimal fractions. Both units are part of the math curriculum for fifth grade in this school. Both classes were taught by the same teacher who served as their regular math teacher.

Instrument

The same test served as pre- and posttest. The test consisted of ten paper and pencil word problems. Three of the items were $x + y = c$ type problems and they served for internal comparisons of students in both groups. Five items were $x/y=c$ problems, and they were taken from different group tests of formal reasoning, as published in professional literature. The other two items were the balance team task, adapted from Lawson's test of formal reasoning (Lowson, 1978), and the paper clip task (Karplus et al, 1974). Both tasks were accompanied by illustrations (The whole test is

shown in Figure 1. Students' responses were scored numerically and qualitatively.

insert Fig. 1. about here

Scoring of Responses

For correct solutions of problesm 1, 4 and 8 (x+y=c type) students received two points for correct solutions, and 0 points for incorrect ones. For all the other problems students received two points for proper use of proportions with correct solution, and one point for correct establishment of proportional relationship without final solution. Solution without recognition of proportional relationship scored 0 points. In addition all responses were assigned to one of the following sub-scale levels of solution:

Level I - No sign of class inclusion

Level II - Class inclusion with additive strategy

Level III - Ration recognition, but no proportional relationship

Level IV - Ratio application to proportional relationships but improper solution

Level V - Proper use of proportion with correct solution

Percentages of responses in each subscale were computed over items, for the experimental and control group separately.

Analysis of Data

When the mean scores of students in the two groups were compared,

an effect size (ES) was calculated by the formula: ES=(Xe-Xc)/Sm, where Xe and Xc are the means of the experimental and control groups respectively, and Sm is the arithmetic mean of the standard deviations of the scores in both groups.

Results and Conclusions

Table I shows the mean scores of students in the experimental and control groups in each of the problems in the pretest.

insert Table I about here

Inspection of the data reveals the following:

a) The two classes were almost equal in their ability to solve items in the pretest.

b) Both classes mastered the x+y=c type problems and had difficulties solving the x/y=c problems.

c) The x/y=c problems were not equal in their level of difficulty. Some of them were easier than the others (see problems 2 and 9, which were answered correctly by about half of the students in both classes).

Table II shows the mean scores of students in the same test, after a treatment period of three weeks. The following can be concluded from Table II data:

insert Table II about here

a) The experimental group performed most of the x/y=c type problems better than the control group (size of effect moved between 0.36 to 2.26 standard deviation units).

b) Both groups did not succeed to solve the balance beam problem.

c) Problems 2 and 9 remained the easiest among the x/y=c problems, even in the post test.

d) Problem 5 was found to be easy for the control group. No sound explanation is found for this data.

Comparison of data in Table I and II indicates that students in the experimental group, after three weeks of learning the subject of ratio and proportions, significantly improved their ability to solve x/y=c type problems. No such improvement was found with the contorl group (except in problem 5, which might be accidental), although they performed the same test the second time. Not only did the experimental group reach the correct solution more often, their responses showed class inclusion perception (realization of the two relationships that exist between the quantities in the problem), ratio recognition and application of the ratio to establish proportional relationship. Correct responses in the control group, for comparison, were based more on intuition and good guesses, as might be concluded from the many cases in which correct answers were not accompanied by numerical computation.

insert Table III about here

Table III summarized the distribution of students' responses to the posttest problems, by the five level scale of solution. It can clearly be seen that while the majority of the students in the control group used the additive strategy to solve the proportion problems, most of the students in the experimental group set the proportional relationship and many of them reached the correct solution.

Discussion

The results of this study showed that learning general procedures of how to solve proportional problems has greatly improved the ability of 10 years old students to solve tasks from different tests of formal reasoning including the paper clip task. This "training" did not help them to successfully cope with the balance bean task, a task which was originally introduced by Inhelder and Piaget (1958) as part of a group of tasks for assessingformal reasoning.

These findings can point out that solutions of the x/y=c type tasks and the original Piagetian task are not based on the same cognitive operations. While the x/y=c tasks can be solved by applying a simple ratio algorithm, the balance beam task requires application of multiplicative compensation of variables in a physical system. As a result, written x/y=c type tasks can be

solved sucessfuly by young students (about 11 years old, in this study), who probably did not develop formal structures of reasoning yet. If this is really the case, x/y=c tasks differentiate masters of proportional tasks' algorithm from non-masters, but not formal from non-formal reasoners. Usually most tests of formal reasoning contain several task of proportional reasoning. The findings of this study call the attention of the users of these tests, to the possible effect which previous learning migh have on the performance of these tests. This effect should be considered when interpretation of students' responses to written tests of formal reasoning is the issue. As failure to solve these tasks is not an indication of lack of formal reasoning ability, sucessful performance is not always an indication of possession of this ability.

This study showed that after a short period of regular math activities, young students can solve many of the proportional reasoning task. On the other hand many studies, including one by the author, found that secondary school and college students do not master the operation of proportional reasoning. What is the cause of this contradictive finding? Undoubtfully forgetfulness of learned algorithm is in the basis of this phenomna. And maybe true formal reasoning is the same group of reasoning patterns that we are left with after we have forgotten all learned school algorithms.

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TABLE I

Means, standard deviations and effect size(es) in the pre-test

Problem No.	Control group n=35	Experimental group n=33	ES
*1	** 1.93 (0.57)	1.88 (0.60)	- 0.08
2	1.09 (0.53)	1.25 (0.54)	+ 0.29
3	1.18 (0.68)	0.11 (0.67)	- 0.10
*4	1.93 (0.52)	1.88 (0.52)	- 0.09
5	0.42 (0.70)	0.28 (0.68)	- 0.20
6	0.18 (0.65)	0.14 (0.63)	- 0.06
7	0.06 (0.34)	0.11 (0.42)	+ 0.13
*8	2.00 (0.00)	1.94 (0.63)	- 0.09
9	1.21 (0.78)	0.87 (0.72)	- 0.45
10	0.06 (0.34)	0.05 (0.46)	- 0.03

* x+y=c type problems.

** Maximal score for each item was 2.

TABLE II

Means, standard deviations and effect size(es) in the pre-test

Problem No.	Control group n=35	Experimental group n=33	ES
*1	1.93 (0.57)	2.00 (0.00)	+ 0.12
2	1.21 (0.68)	1.71 (0.56)	+ 0.80
3	0.36 (0.66)	1.27 (0.64)	+ 1.38
*4	2.00 (0.00)	1.94 (0.58)	- 0.10
5	1.45 (0.56)	1.60 (0.62)	+ 0.53
6	0.42 (0.71)	1.65 (0.57)	+ 1.60
7	0.06 (0.35)	0.29 (0.43)	+ 0.36
*8	2.00 (0.00)	1.94 (0.58)	- 0.11
9	1.06 (0.54)	1.88 (0.61)	+ 1.08
10	0.06 (0.35)	1.57 (0.64)	+ 2.26

* x+y=c type problems.

TABLE III

Distribution of responses to the post-test, by type of solution strategy (in %)

*Solution strategy	Control group	Experimental group
Level I	7	3
Level II	52	12
Level III	20	6
Level IV	11	32
Level V	10	47

* Solution strategies in each level are explained in the method part.

1) David read 22 pages of his book. He has to read 50 pages more to finish it. How many pages are in David's book?

2) Six small pieces of wood are equal in length to four large pieces of wood. How many small pieces are equal in length to six large pieces of wood?

3) The "Bears" won five out of their first seven games. If they keep winning at the same rate, how many games will they have played before they win fifteen games?

4) When David was four years old, Tami, his sister, was six years old. How old will Tami be when David is six years old?

5) Tami can read 22 pages in 50 minutes. If she continues to read at the same rate, how long will it take her to read 99 pages?

6) Two trees are casting shadows on the playground. The three metre tree casts a shadow five metres long. How long will the shadow of the five metre tree be?

7) Suppose you have to balance the beam (see picture). You have a five gram weight. At what distance from the pivot point do you have to put it to balance a seven gram weight, that is ten centimetres away from the pivot point?

8) Ronit had two pencils. She received a new pack of ten pencils and she gave her friend five of them. How many pencils has Ronit left?

9) To bake four cakes you need ten eggs. How many eggs would you need to bake six cakes using the same recipe?

10) The height of Mr. Short, when it is measured with buttons, is 4 buttons and Mr. Tall's height is 6 buttons. When measuring is done with paper clips, Mr. Short's height is 6 paper clips. What is the height of Mr. Tall in paper clips?

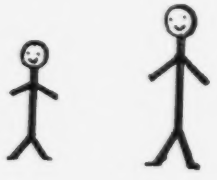

Figure Caption

Figure 1: Test items